A non-parametric regression model for estimation of ionospheric plasma velocity distribution from SuperDARN data

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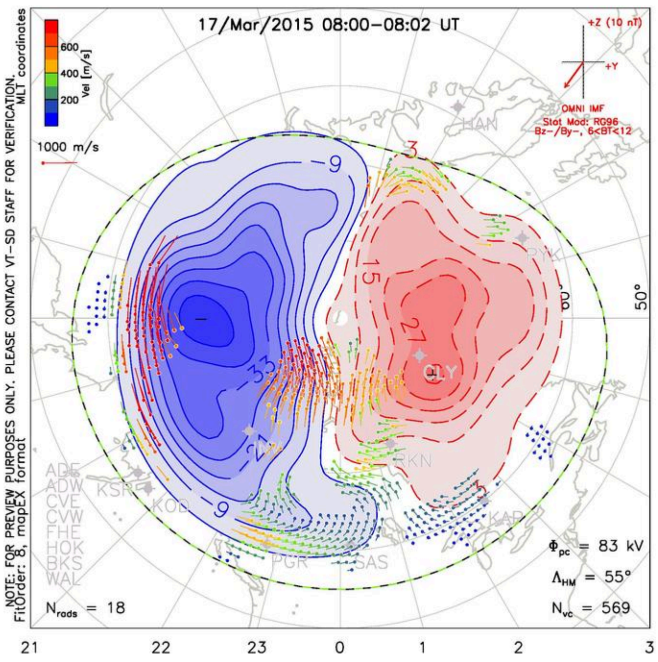
Ionospheric convection pattern

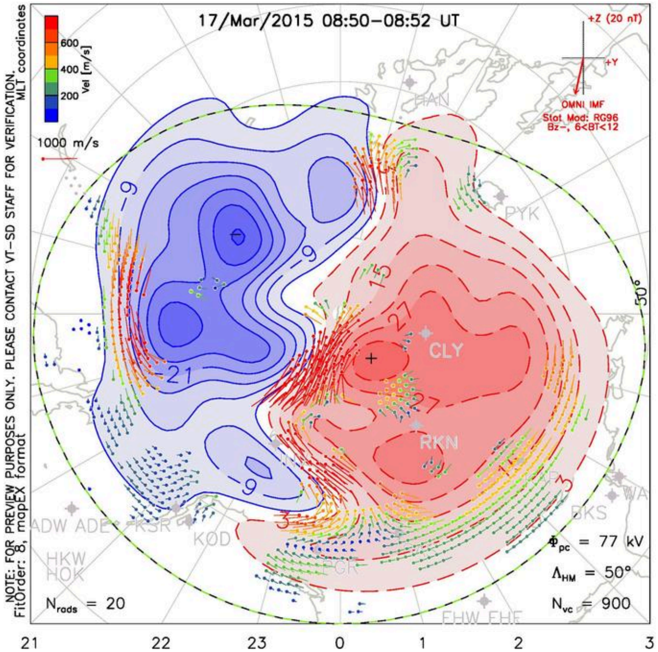
- The ionospheric flow pattern is one of fundamental property of the ionospheric science.
- There exist several studies which deduced the velocity distribution pattern in the ionosphere.



Ionospheric convection pattern deduced by the spherical harmonics method (http://vt.superdarn.org/ Virginia Tech Website)

Existing methods

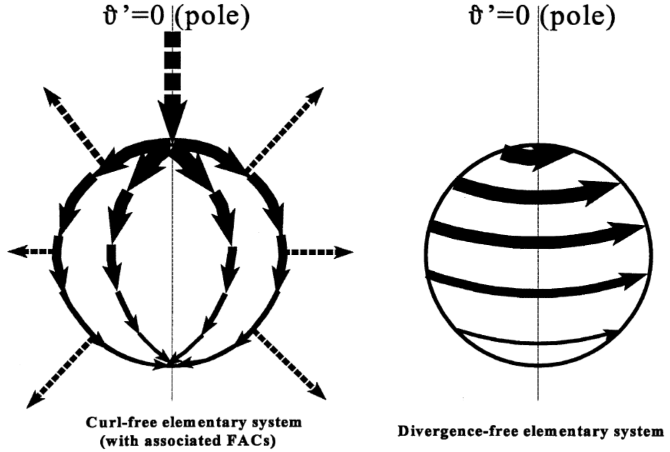
- Spherical harmonic fitting
  - Assuming that the divergence-free condition, the vector field can be represented by a scalar stream function. The stream function is expanded with spherical harmonics funcitons.
  - Sensitive to local distrubances and noises
- Matrix-valued kernel (Narcowich et al., 2007; Fuselier and Wright, 2009)
  - Localized basis function
  - Computationally demanding
  - Designed for interpolating vector-valued data
- Spherical elementary current system (Amm, 1997)
  - Localized basis function
  - Diverge at the singular point of a basis function



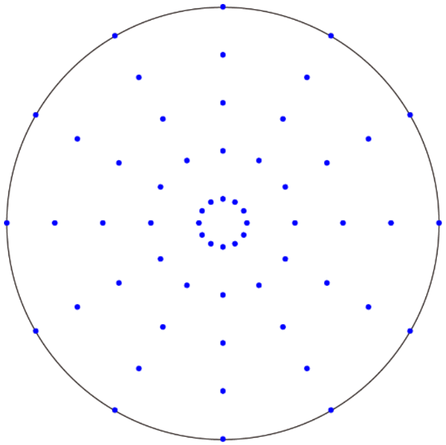
Ionospheric convection pattern deduced by the spherical harmonics method (http://vt.superdarn.org/ Virginia Tech Website)

Spherical elementary current system (SECS)

Curl-free element: $\boldsymbol{J}_{cf} = \dfrac{I_{cf}}{4\pi R_I}\cot\left(\dfrac{\theta'}{2}\right)\hat{\boldsymbol{e}}_{\theta'}$, Divergence-free element: $\boldsymbol{J}_{df} = \dfrac{I_{df}}{4\pi R_I}\cot\left(\dfrac{\theta'}{2}\right)\hat{\boldsymbol{e}}_{\phi'}$



Sketch of spherical elementary current systems (SECS) (Amm and Vilijanen, 1999)



An example of node distribution

- The divergence-free SECS basis function is defined so that the curl is constant except at the pole, and the curl-free SECS basis function is defined so that the divergence is constant except at the pole.
- Nodes of the SECS basis functions can be placed arbitrarily. This can be regarded as one of radial basis function (RBF) networks.
- They diverge to infinity at the pole.

Generalization of SECS

- The ionospheric plasma drift velocity distribution can be assumed to be divergence-free (no source, no sink).
- The divergence-free vector field can be represented by a stream function Ψ as follows:

$$\boldsymbol{V}(\boldsymbol{r}) = -\hat{\boldsymbol{r}} \times \nabla \Psi.$$

- We expand the stream function Ψ by using localized basis functions:

$$\Psi(\boldsymbol{r}) = \sum_i w_i \psi(\boldsymbol{r}, \boldsymbol{r}_i).$$

- Thus,

$$\boldsymbol{V}(\boldsymbol{r}) = -\hat{\boldsymbol{r}} \times \nabla \Psi = -\sum_i w_i \hat{\boldsymbol{r}} \times \nabla \psi(\boldsymbol{r}, \boldsymbol{r}_i).$$

Generalization of SECS

- Defining vector-valued localized basis function:

$$\boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i) = -\boldsymbol{e}_r \times \nabla\psi(\boldsymbol{r},\boldsymbol{r}_i),$$

we obtain

$$\boldsymbol{V}(\boldsymbol{r}) = \sum_i w_i \boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i).$$

- If $\psi(\boldsymbol{r},\boldsymbol{r}_i) = 2\log\left|\sin\frac{\Delta\theta'}{2}\right|$ where $\theta' = \arccos\left(\frac{\boldsymbol{r}\cdot\boldsymbol{r}_i}{R^2}\right),$

$$\boldsymbol{v}(\boldsymbol{r},\boldsymbol{r}_i) = \boldsymbol{e}_{\phi,i}\cot\frac{|\Delta\theta'|}{2}.$$

This is the original divergence-free SECS basis function.

Generalization of SECS

- We represent the divergence-free velocity field by

$$\boldsymbol{V}(\boldsymbol{r}) = \sum_i w_i \boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i), \quad \left(\boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i) = -\hat{\boldsymbol{r}} \times \nabla \psi(\boldsymbol{r}, \boldsymbol{r}_i)\right).$$

- We choose the spherical Gaussian function for ψ:

$$\psi(\boldsymbol{r}, \boldsymbol{r}_i) = \exp\left[\eta\left(\frac{\boldsymbol{r} \cdot \boldsymbol{r}_i}{R_I^2} - 1\right)\right] = \exp\left[\eta\left(\cos\theta' - 1\right)\right],$$

and obtain the following divergence-free basis function

$$\boldsymbol{v}(\boldsymbol{r}, \boldsymbol{r}_i) = (\eta \boldsymbol{r}_i \times \boldsymbol{r}) \exp\left[\eta\left(\frac{\boldsymbol{r} \cdot \boldsymbol{r}_i}{R_I^2} - 1\right)\right].$$

Kalman filter

- We assume the temporal evolution of the weights $\boldsymbol{w}$

$$p(\boldsymbol{w}_k \mid \boldsymbol{w}_{k-1}) = \mathrm{N}\,(\alpha \boldsymbol{w}_{k-1}, \mathrm{Q}).$$

- The residual component can then be estimated with the following Kalman filter algorithm:

Prediction:

$$\boldsymbol{w}_{k|k-1} = \alpha \boldsymbol{w}_{k-1|k-1},$$

$$\mathrm{P}_{k|k-1} = \alpha^2 \mathrm{P}_{k-1|k-1} + \mathrm{Q}.$$

Filtering:

$$\boldsymbol{w}_{k|k} = \boldsymbol{w}_{k|k-1} + (\mathrm{P}_{k|k-1}^{-1} + \mathrm{H}_k^T \mathrm{R}_k^{-1} \mathrm{H}_k)^{-1} \mathrm{H}_k^T \mathrm{R}_k^{-1} (\boldsymbol{y}_k - \mathrm{H}_k \boldsymbol{w}_k),$$

$$\mathrm{P}_{k|k} = (\mathrm{P}_{k|k-1}^{-1} + \mathrm{H}_k^T \mathrm{R}_k^{-1} \mathrm{H}_k)^{-1}.$$

Estimation

- The covariance matrices Q is given so as to satisfy

$$\boldsymbol{w}_k^T \mathrm{Q}^{-1} \boldsymbol{w}_k = \sigma_Q^2 \sum_i \left\{ \left[\frac{1}{\cos\lambda} \frac{\partial}{\partial\lambda} \left(\cos\lambda \frac{\partial w_k}{\partial\lambda} \right) \right]_i + \left[\frac{\exp(\xi \cos^2\lambda)}{\cos^2\lambda} \frac{\partial^2 w_k}{\partial\phi^2} \right]_i \right\}^2 .$$

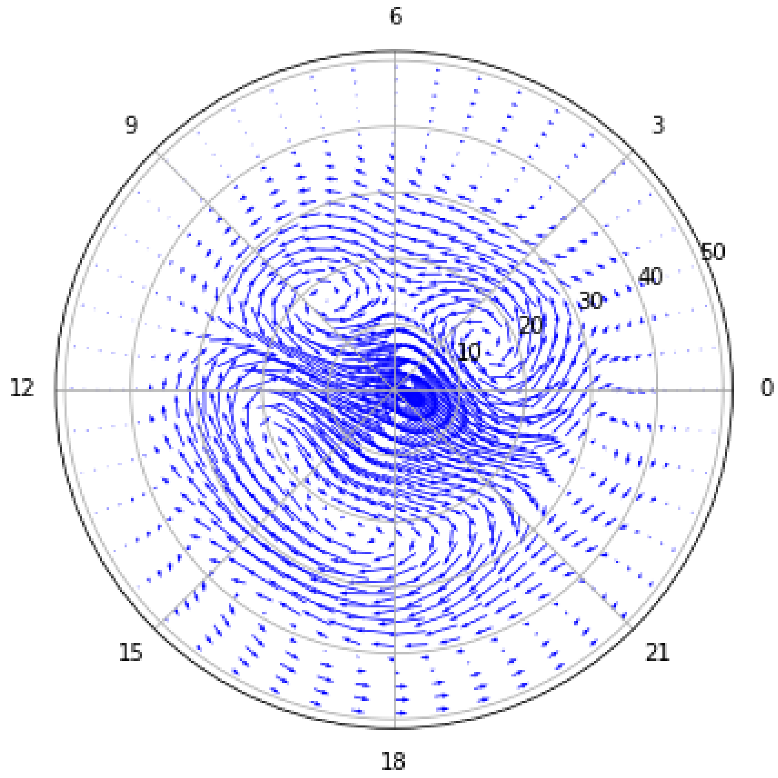
- And R is assumed to be diagonal: $\mathrm{R}_k = \sigma_R^2 \mathrm{I}, \quad (\sigma_R = 500).$
- The parameters σ_Q and ξ are determined by maximizing the marginal likelihood:

$$p(\boldsymbol{y}_{1:K} \mid \boldsymbol{\theta}) = \prod_k \int p(\boldsymbol{y}_k \mid \boldsymbol{w}_k)\, p(\boldsymbol{w}_k \mid \boldsymbol{\theta})\, d\boldsymbol{w}_k .$$

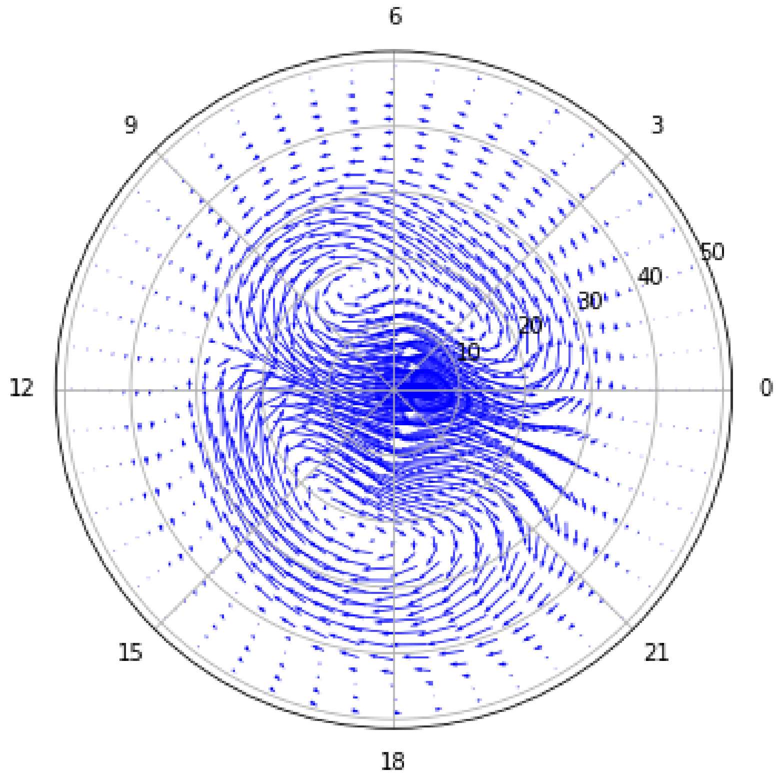
Experiment

- We conducted experiments with synthetic radar data generated from a certain velocity distribution model.
- The observation sites and observed echoes were assumed to be the same as observed on March 17, 2015.
- The nodes of the basis functions were placed at every 5 and 2 degrees in longitude and in latitude, respectively.

Reconstruction with proposed functions



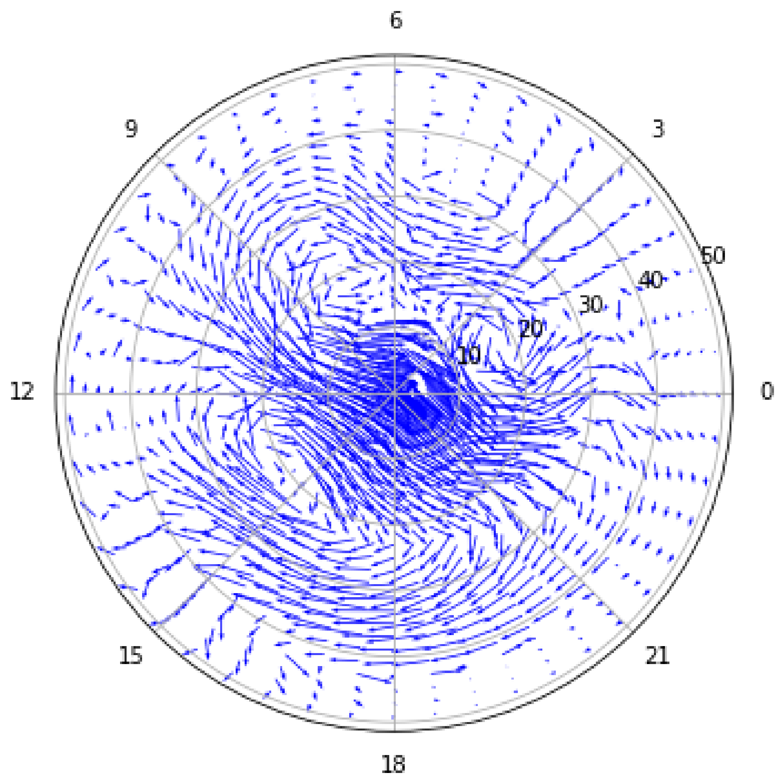
Estimated velocity distribution



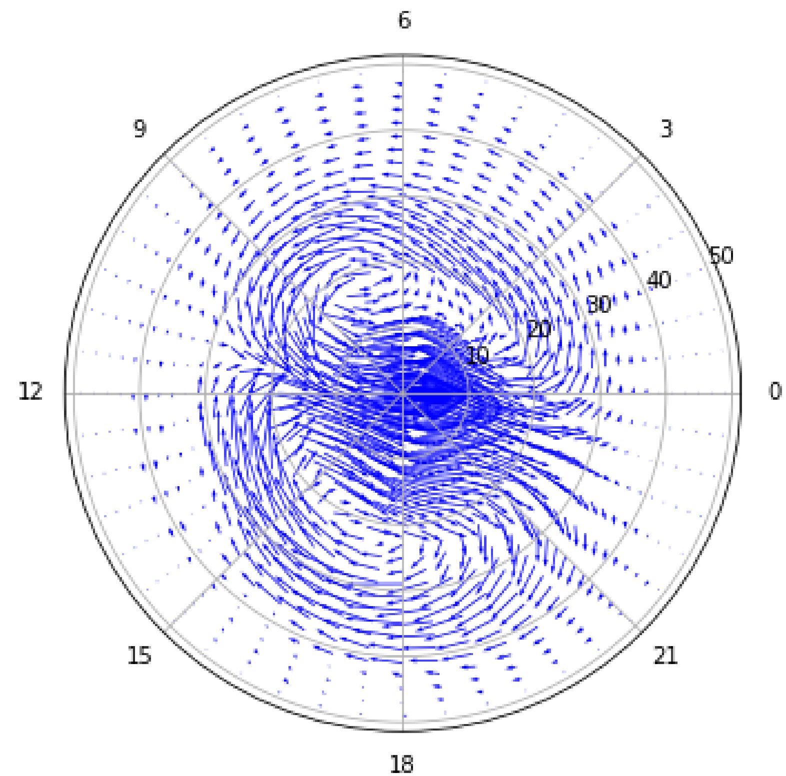
Original velocity distribution

0800 UT, Mar. 17, 2015

Reconstruction with SECS functions



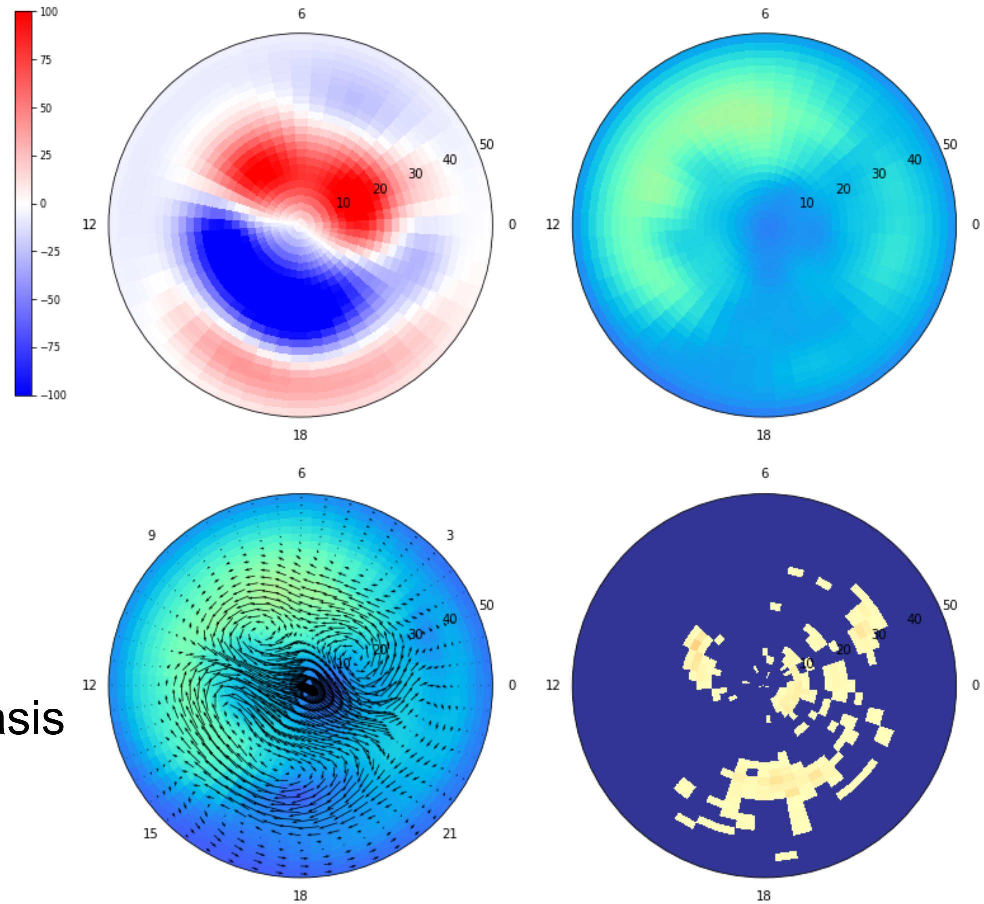
Estimated velocity distribution



Original velocity distribution

0800 UT, Mar. 17, 2015

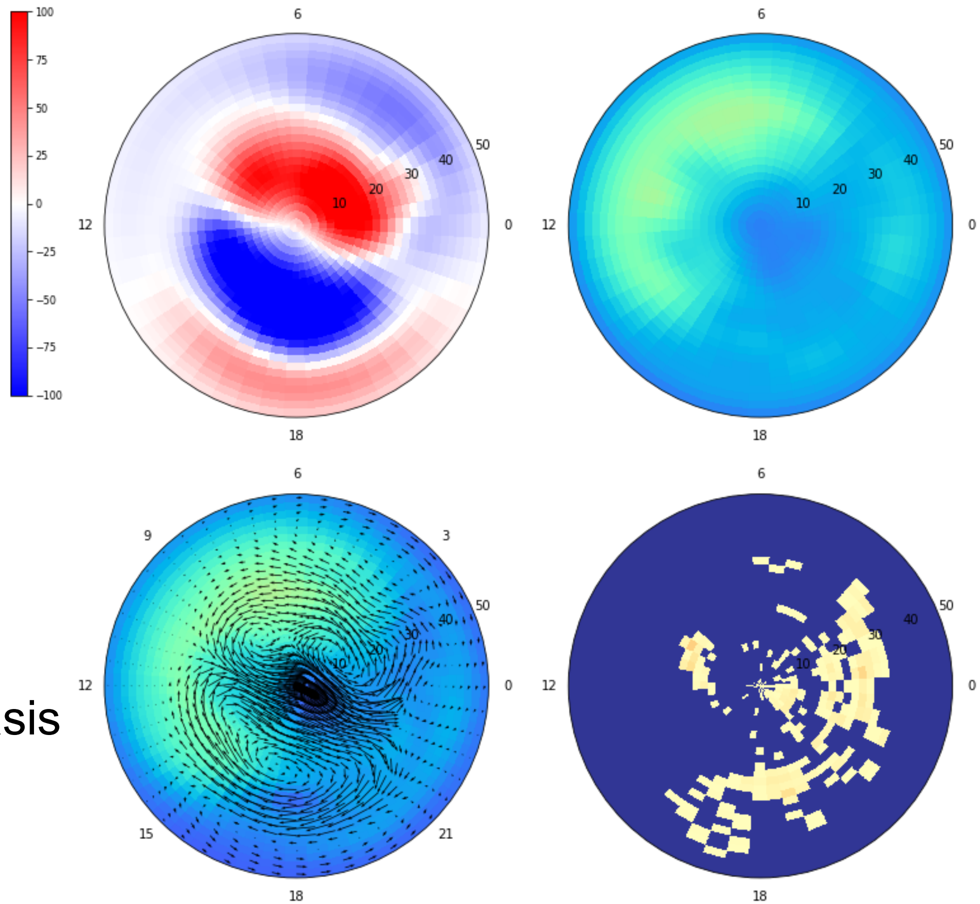
Stream function with proposed



The estimate with proposed basis funcitons.

0800 UT, Mar. 17, 2015

Stream function with proposed



The estimate with proposed basis funcitons.

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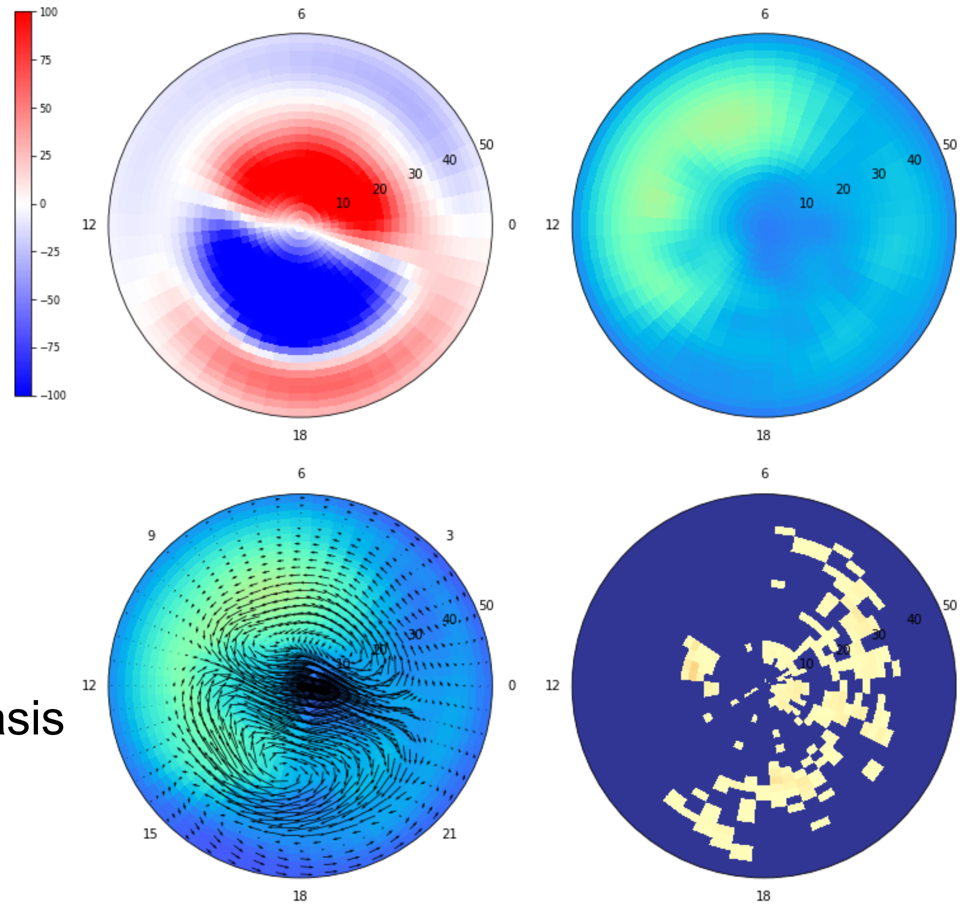
Stream function with proposed



The estimate with proposed basis funcitons.

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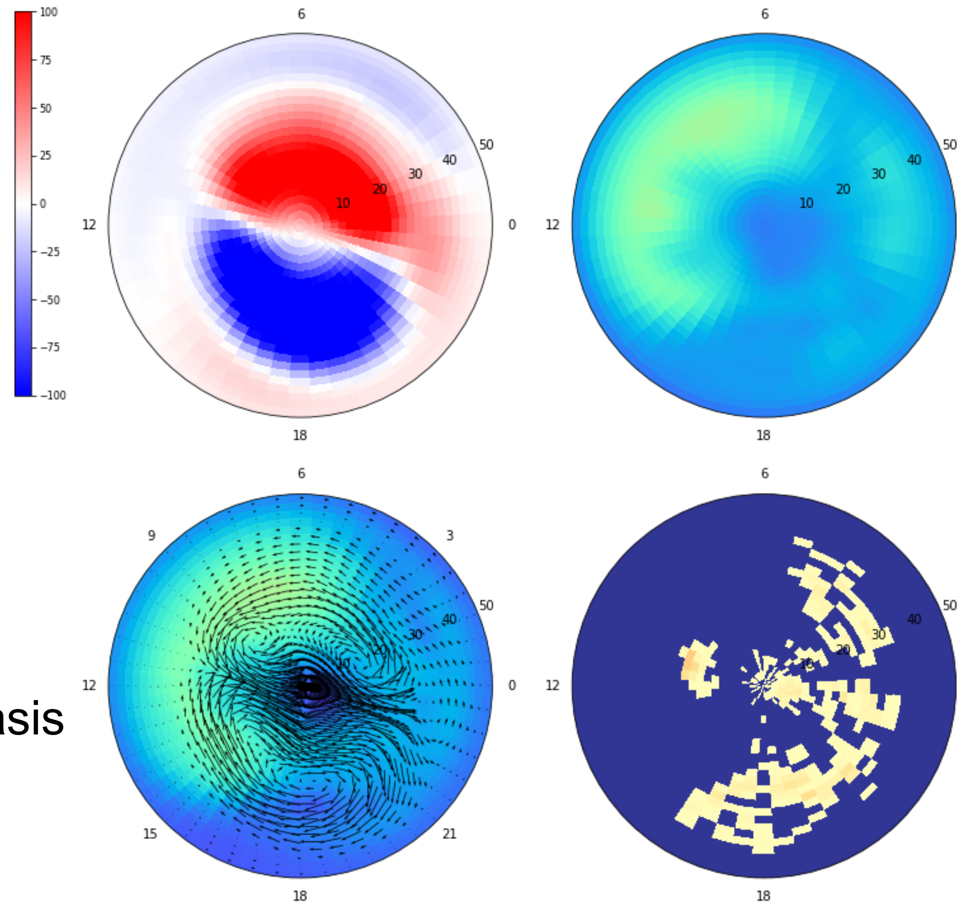
Stream function with proposed



The estimate with proposed basis funcitons.

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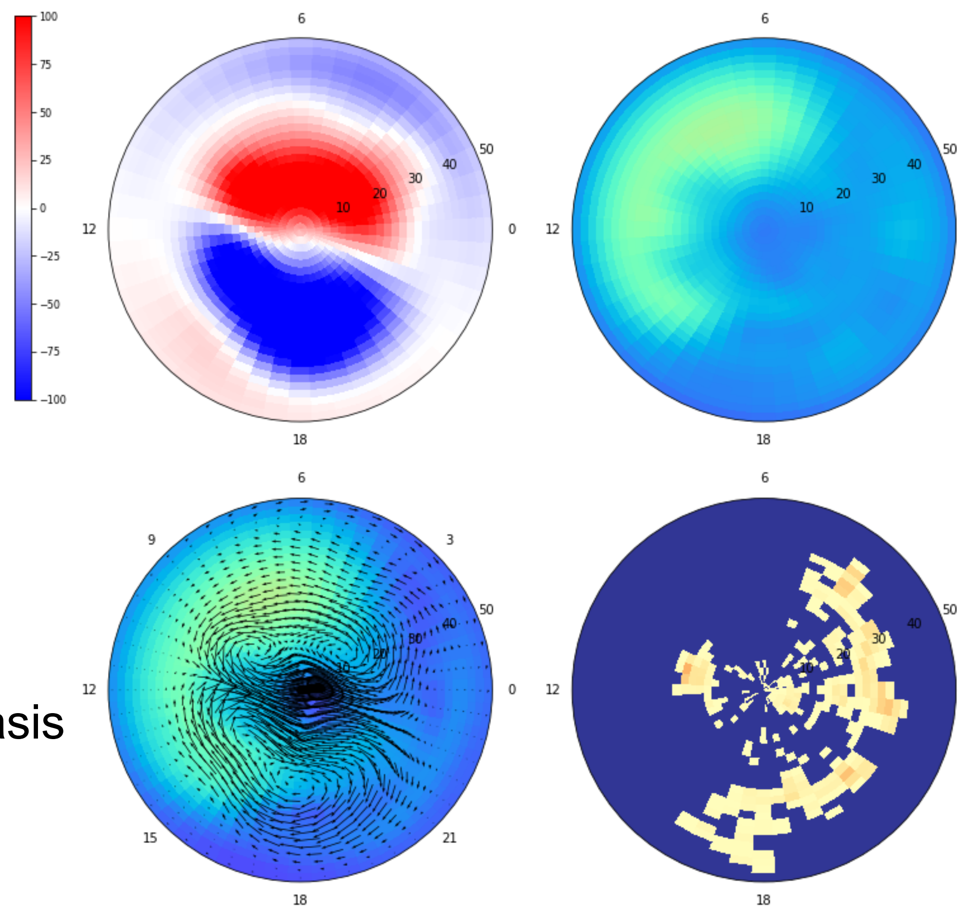
Stream function with proposed



The estimate with proposed basis funcitons.

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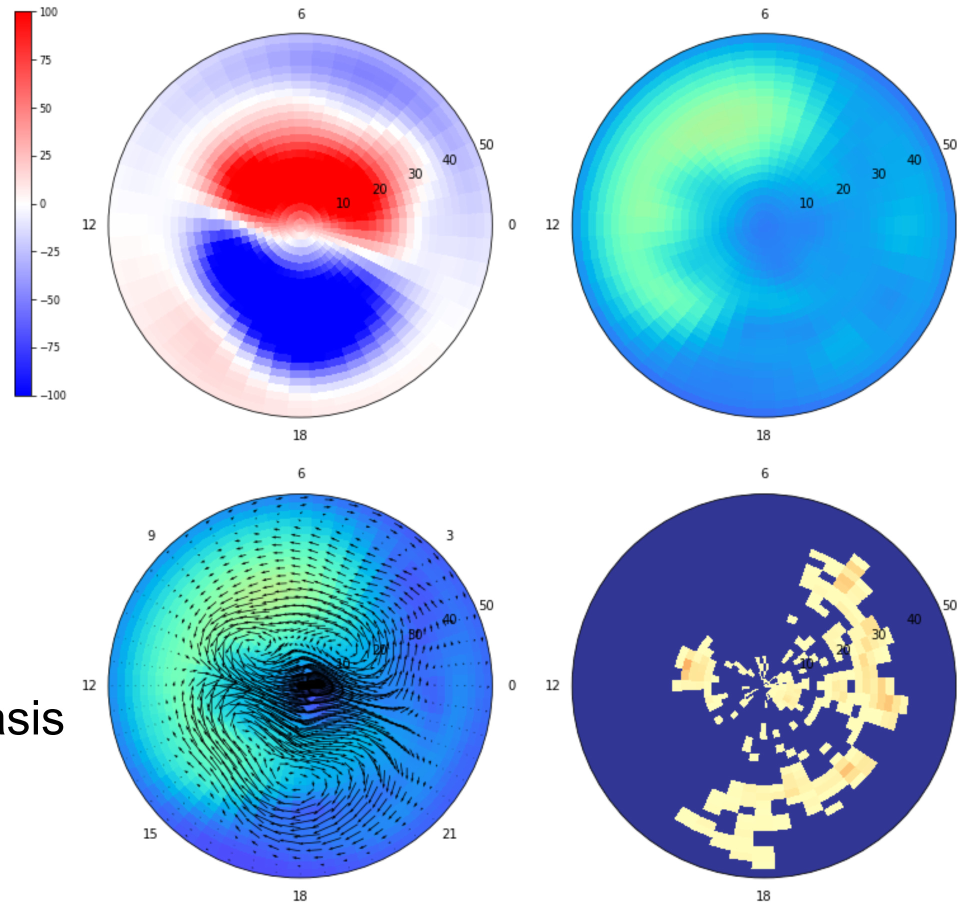
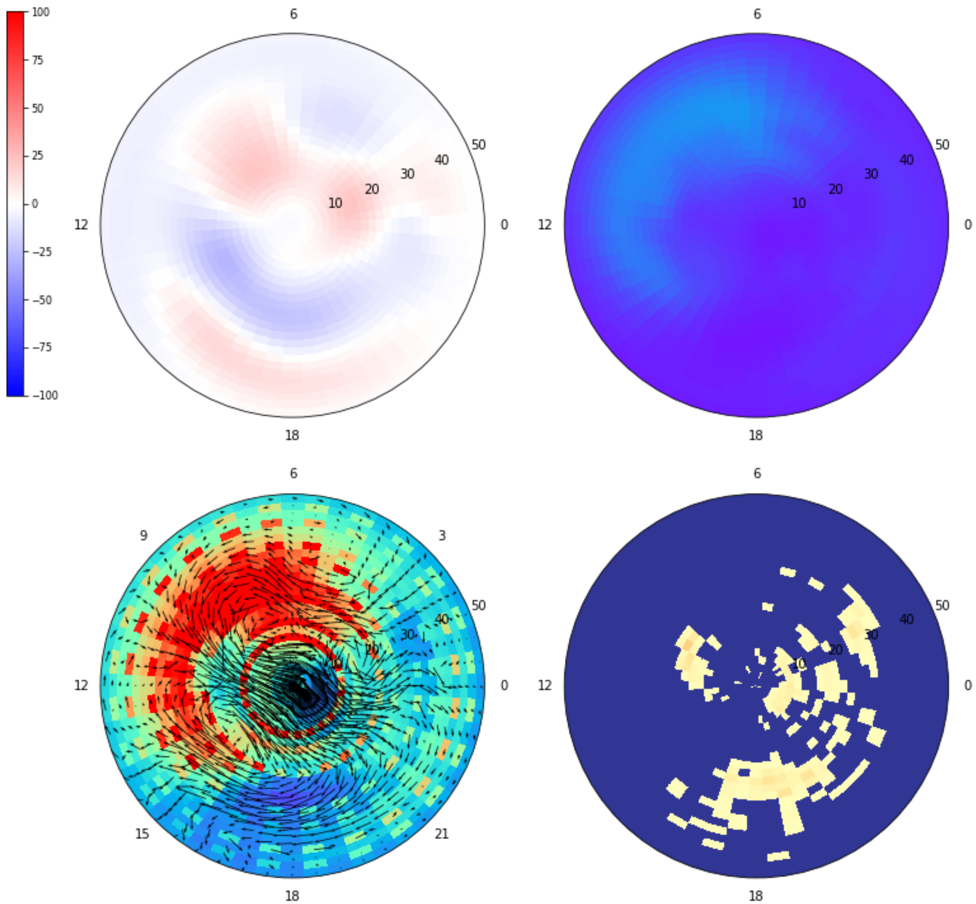
Stream function with proposed



The estimate with proposed basis funcitons.

0850 UT, Mar. 17, 2015

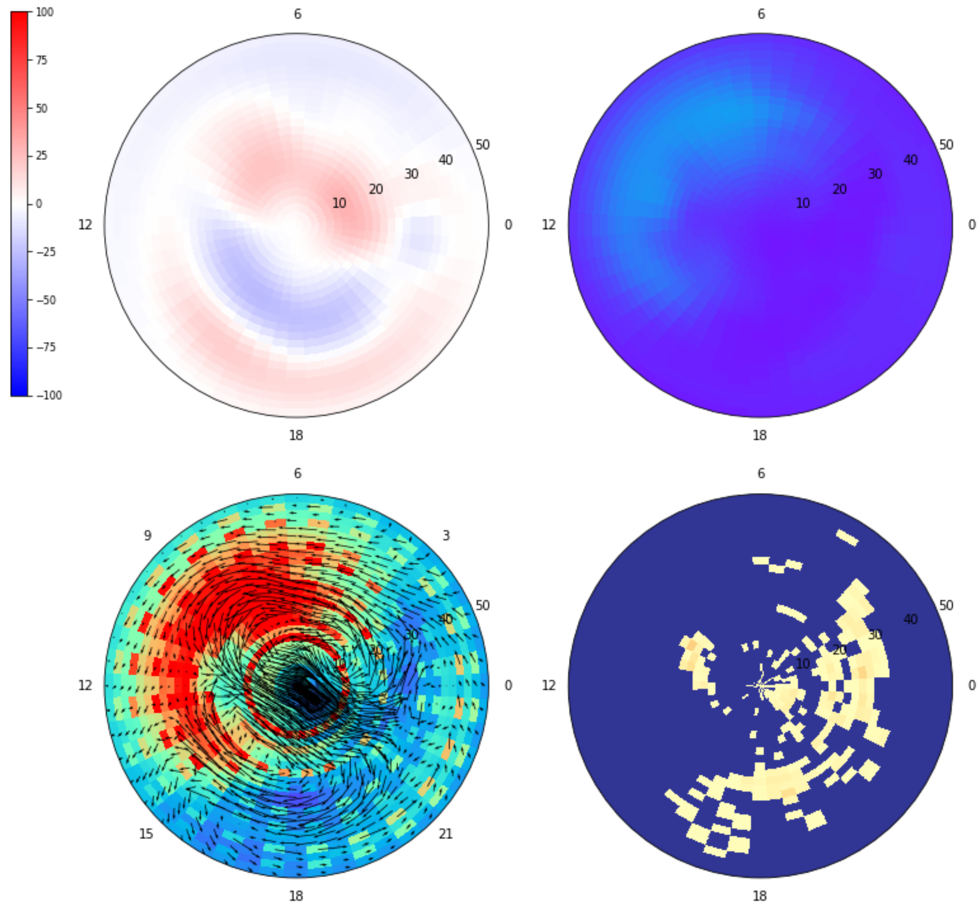
Stream function with SECS



The estimate with SECS basis funcitons.

0800 UT, Mar. 17, 2015

Stream function with SECS



The estimate with SECS basis funcitons.

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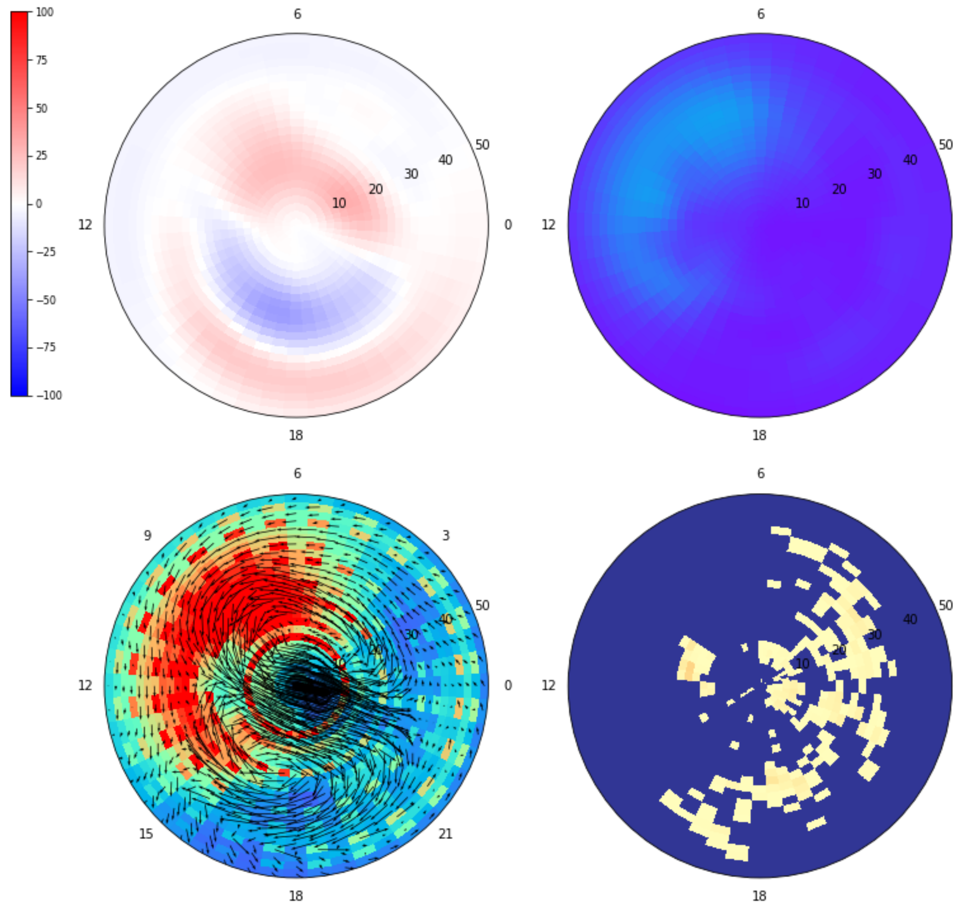
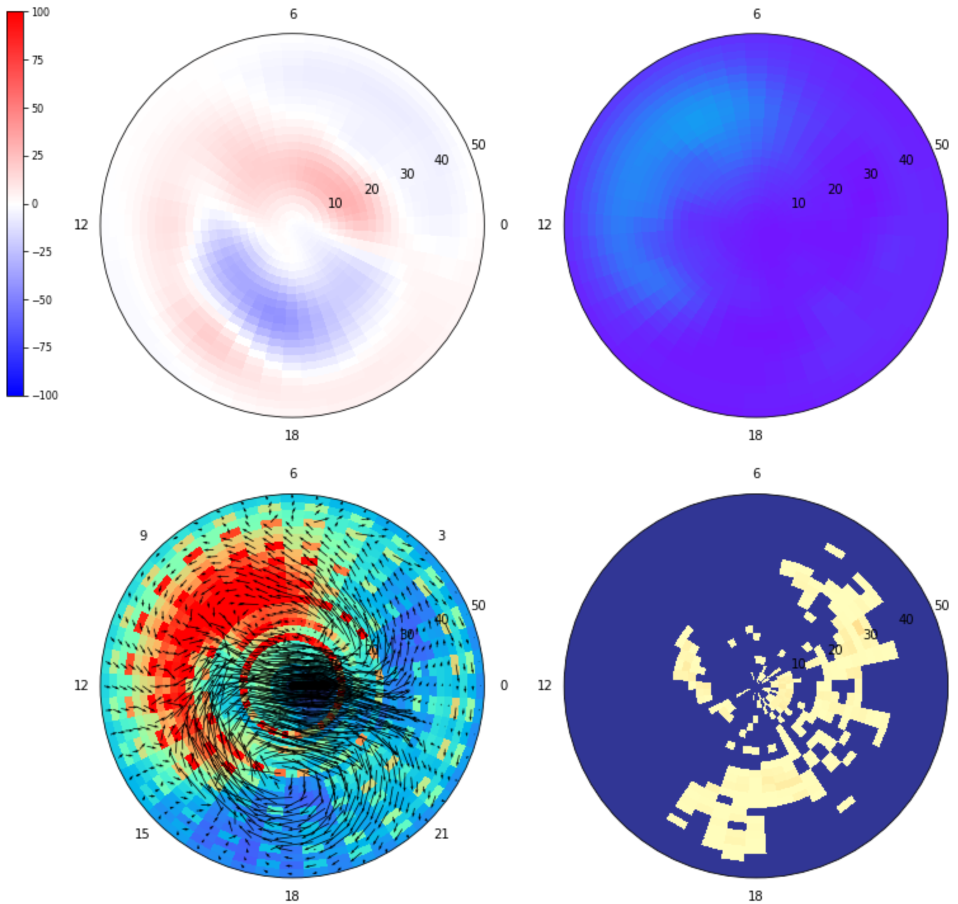
Stream function with SECS



The estimate with SECS basis funcitons.

0820 UT, Mar. 17, 2015

Stream function with SECS



The estimate with SECS basis funcitons.

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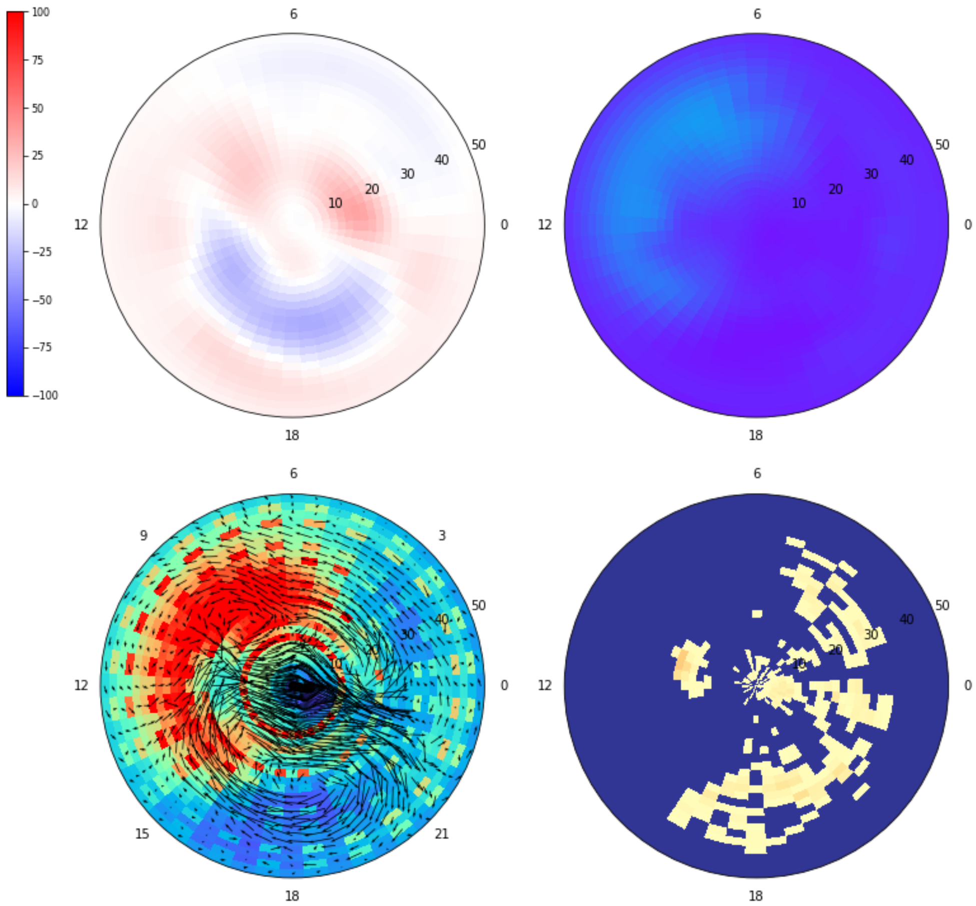
Stream function with SECS



The estimate with SECS basis funcitons.

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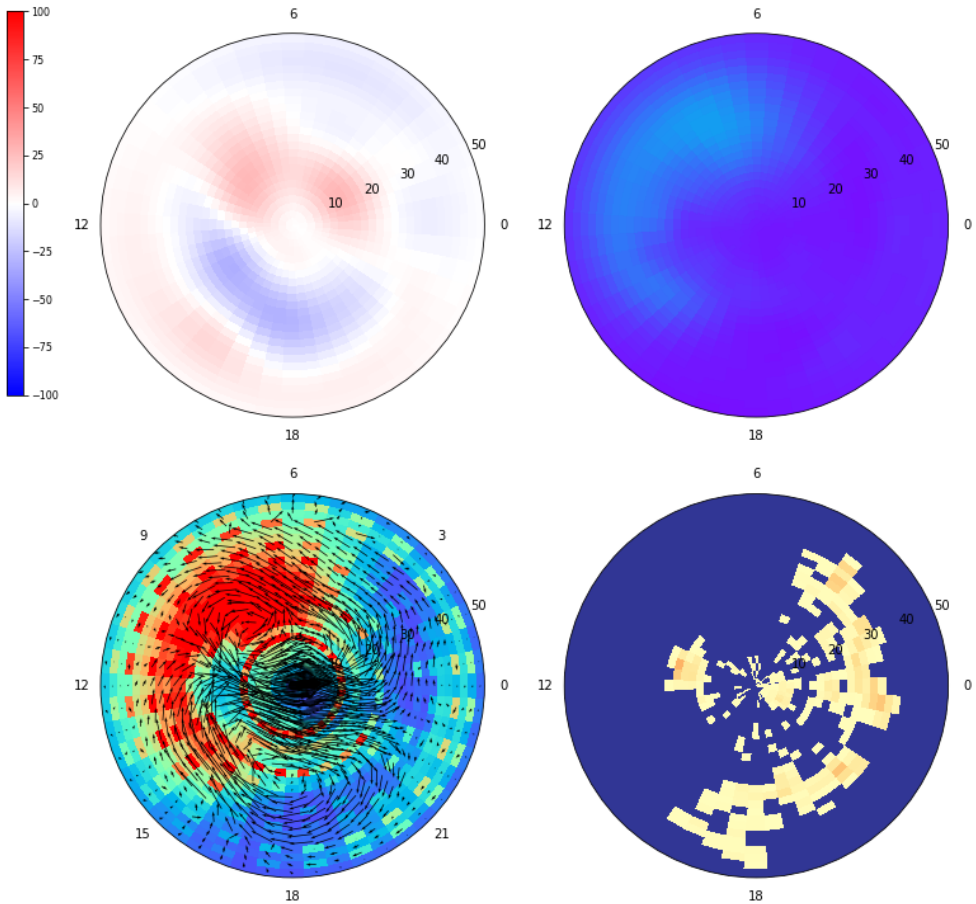
Stream function with SECS



The estimate with SECS basis funcitons.

0850 UT, Mar. 17, 2015

Use of empirical model

- An empirical model is referred to in estimating the velocity distribution.
- We assume the weight $\boldsymbol{w}$ can be decomposed into the model-based value ζ and the residual $\boldsymbol{\beta}$:

$$\boldsymbol{w} = \zeta + \boldsymbol{\beta}.$$

- The model-based value is determined so as to fit an empirical model by Weimber 2001.
- The residual $\boldsymbol{\beta}$ is estimated with the Kalman filter.

Kalman filter

- We assume the temporal evolution of the resudial component obeys

$$p(\boldsymbol{\beta}_k \mid \boldsymbol{\beta}_{k-1}) = \mathrm{N}\,(\alpha \boldsymbol{\beta}_{k-1}, \mathrm{Q}).$$

- The residual component can then be estimated with the following Kalman filter algorithm:

Prediction:

$$\boldsymbol{\beta}_{k|k-1} = \alpha \boldsymbol{\beta}_{k-1|k-1},$$

$$\mathrm{P}_{k|k-1} = \alpha^2 \mathrm{P}_{k-1|k-1} + \mathrm{Q}.$$

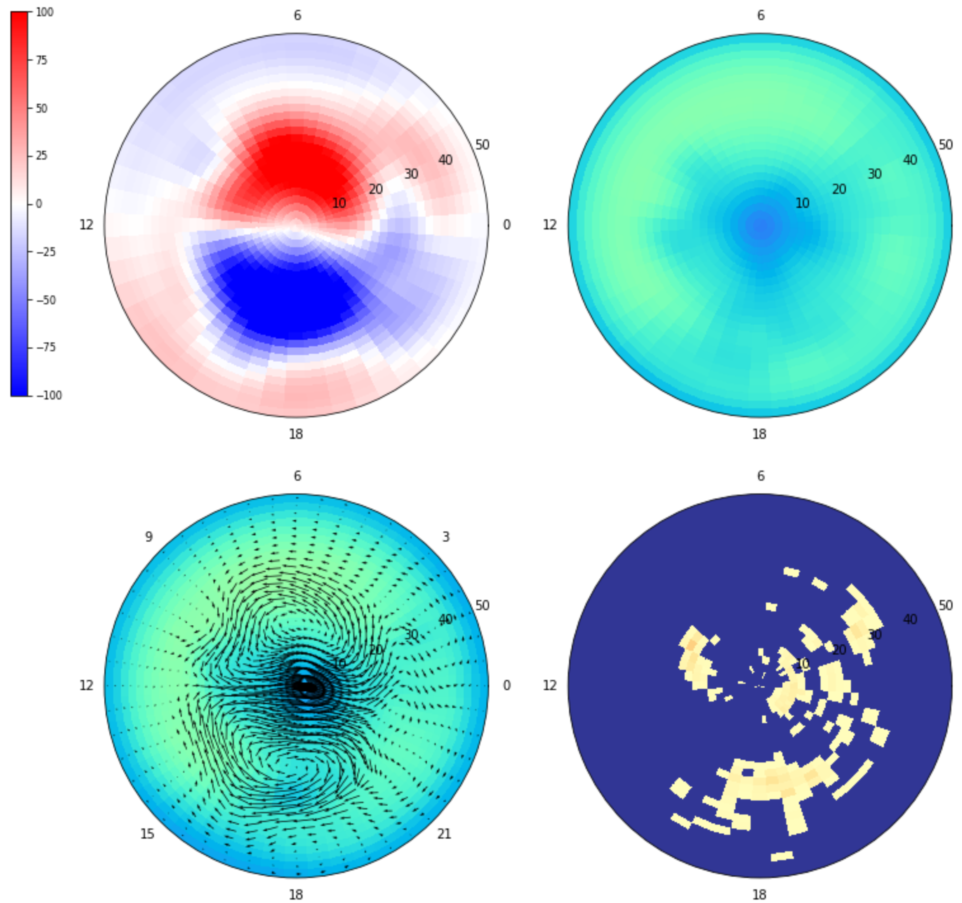
Filtering:

$$\boldsymbol{\beta}_{k|k} = \boldsymbol{\beta}_{k|k-1} + \mathrm{P}_{k|k-1}\mathrm{H}_k^T(\mathrm{H}_k\mathrm{P}_{k|k-1}\mathrm{H}_k^T + \mathrm{R}_k)^{-1}(\boldsymbol{y}_k - \mathrm{H}_k[\boldsymbol{\zeta}_k + \boldsymbol{\beta}_k]),$$

$$\mathrm{P}_{k|k} = \mathrm{P}_{k|k-1} - \mathrm{P}_{k|k-1}\mathrm{H}_k^T(\mathrm{H}_k\mathrm{P}_{k|k-1}\mathrm{H}_k^T + \mathrm{R}_k)^{-1}\mathrm{H}_k\mathrm{P}_{k|k-1}.$$

17 March 2015

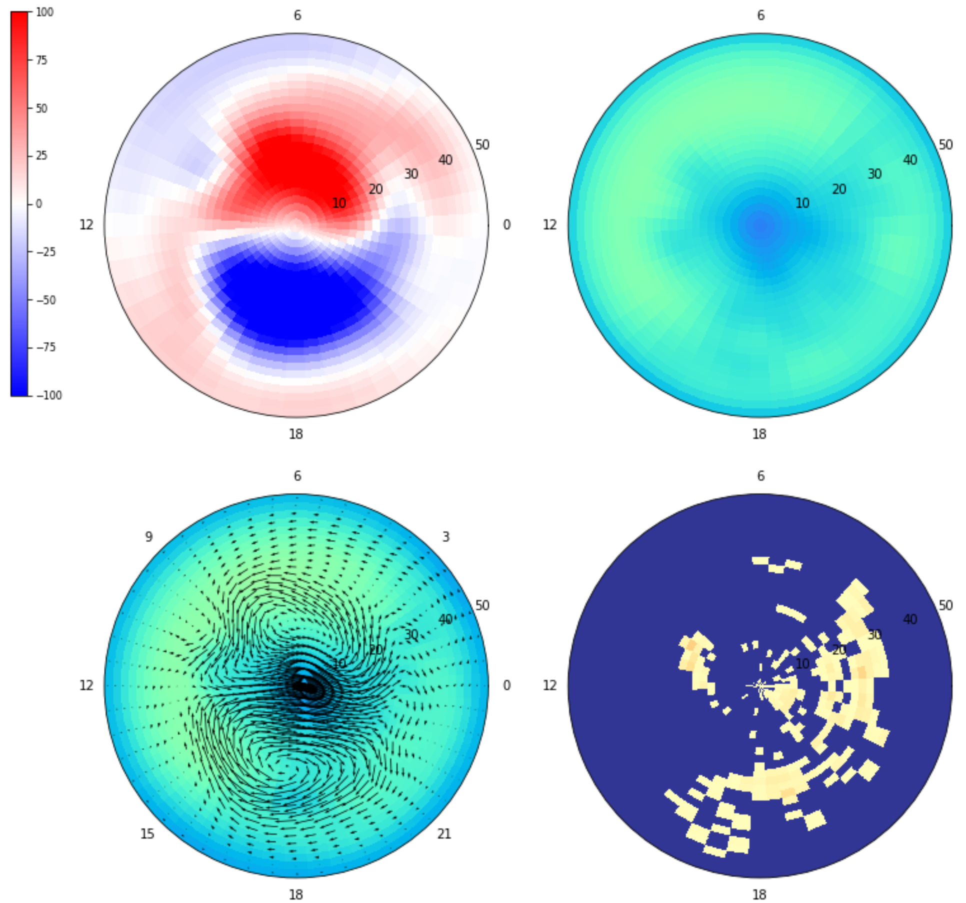
Result



The estimate with actual data

0800 UT, Mar. 17, 2015

Result

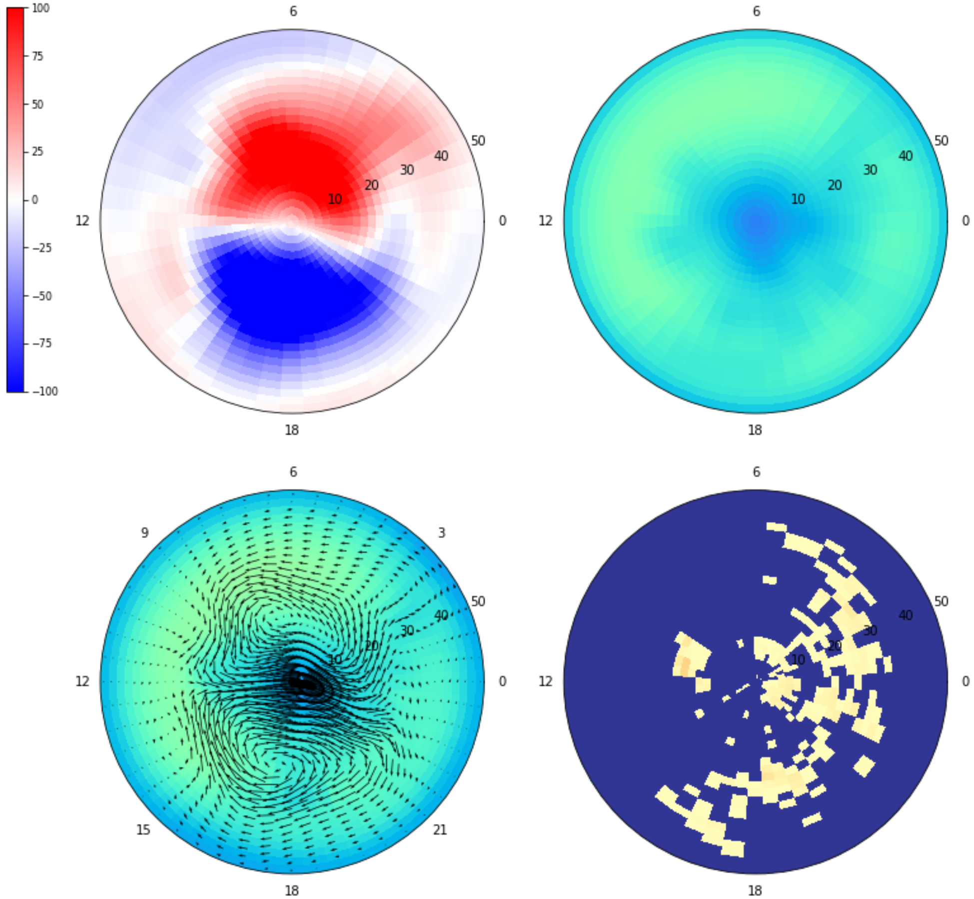


The estimate with actual data

0810 UT, Mar. 17, 2015

Result

The estimate with actual data



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Result

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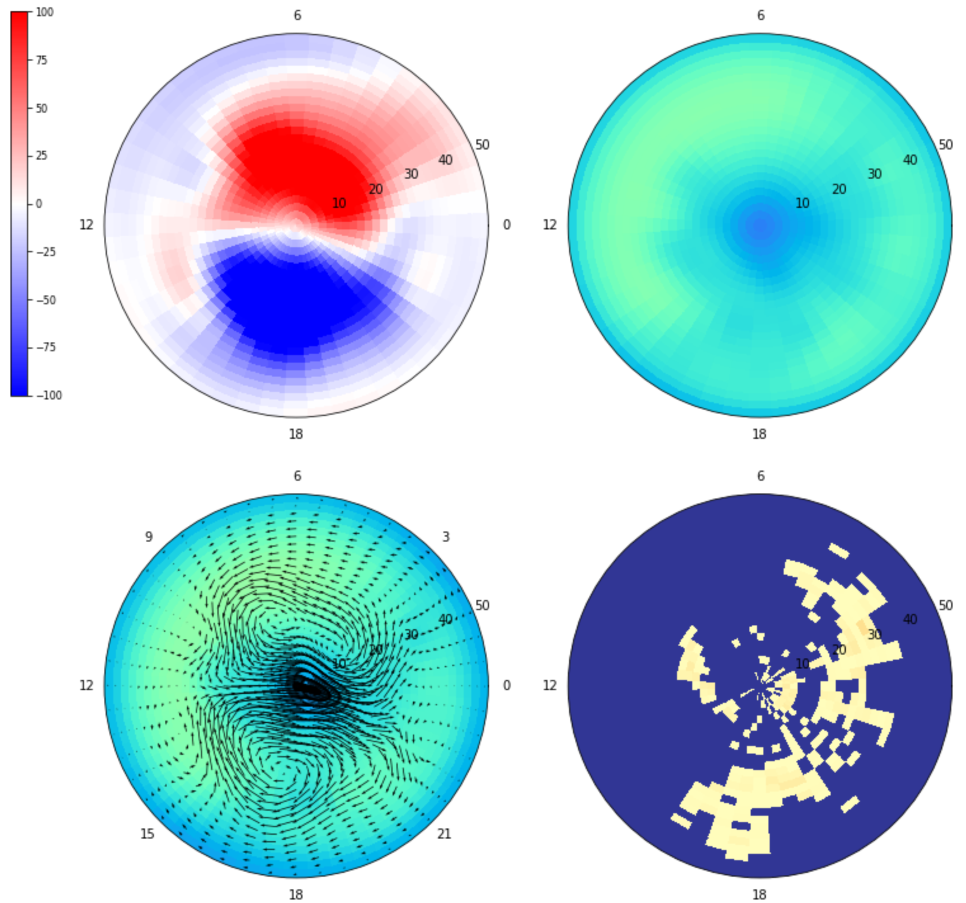

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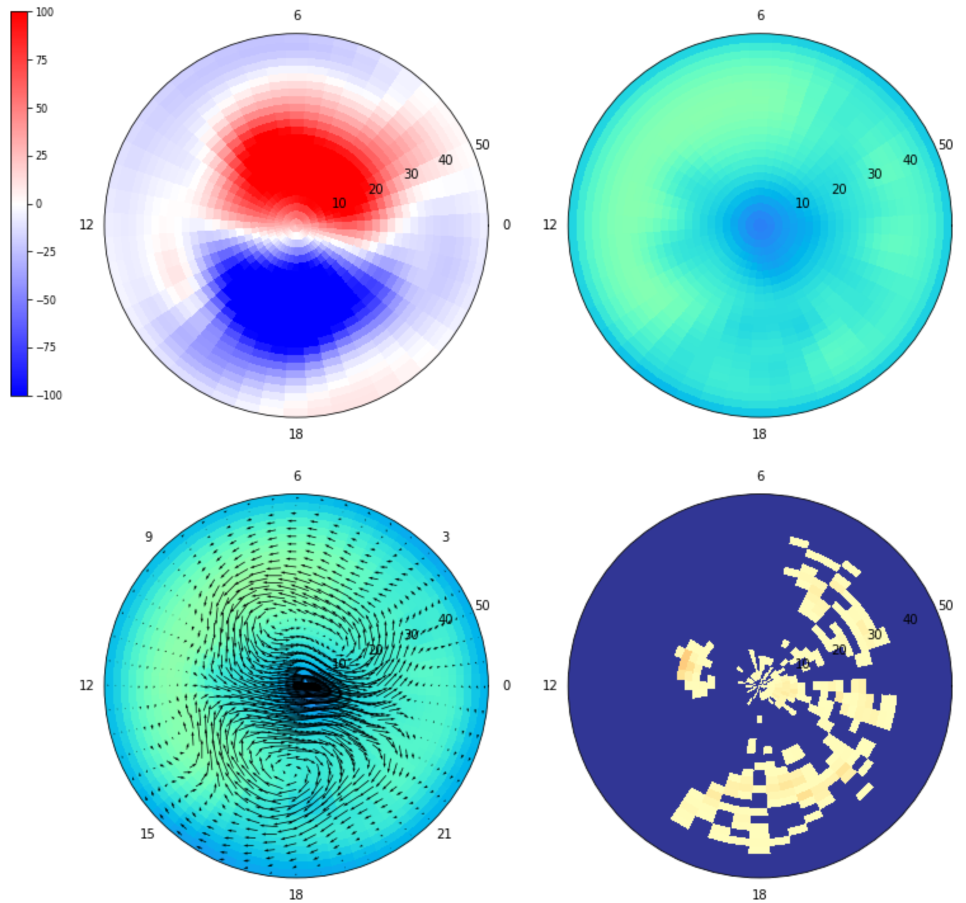
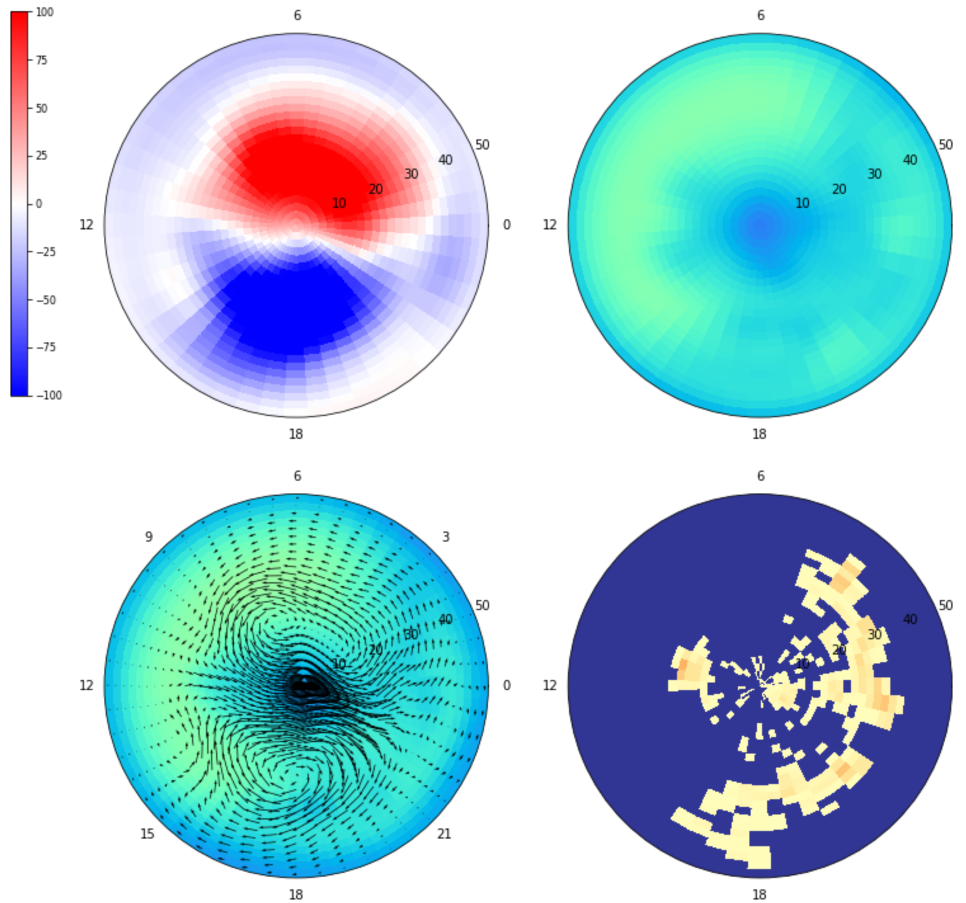
Result



The estimate with actual data

0840 UT, Mar. 17, 2015

Result

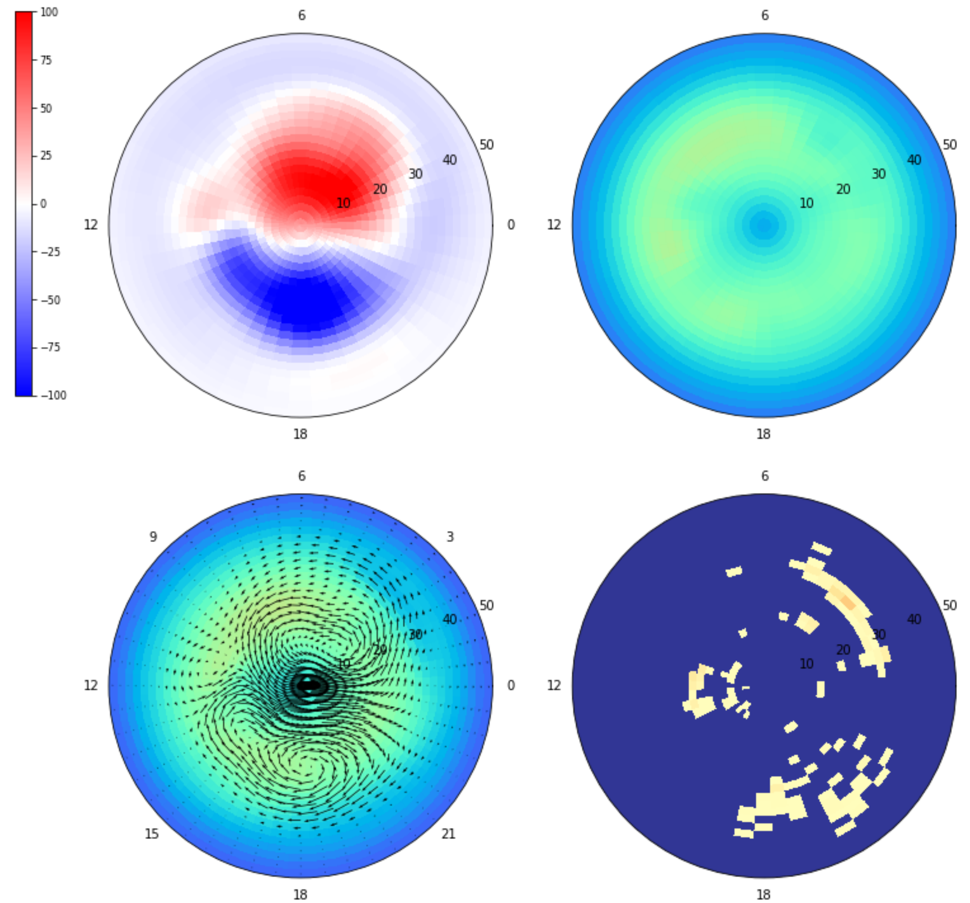


The estimate with actual data

0850 UT, Mar. 17, 2015

27 March 2017

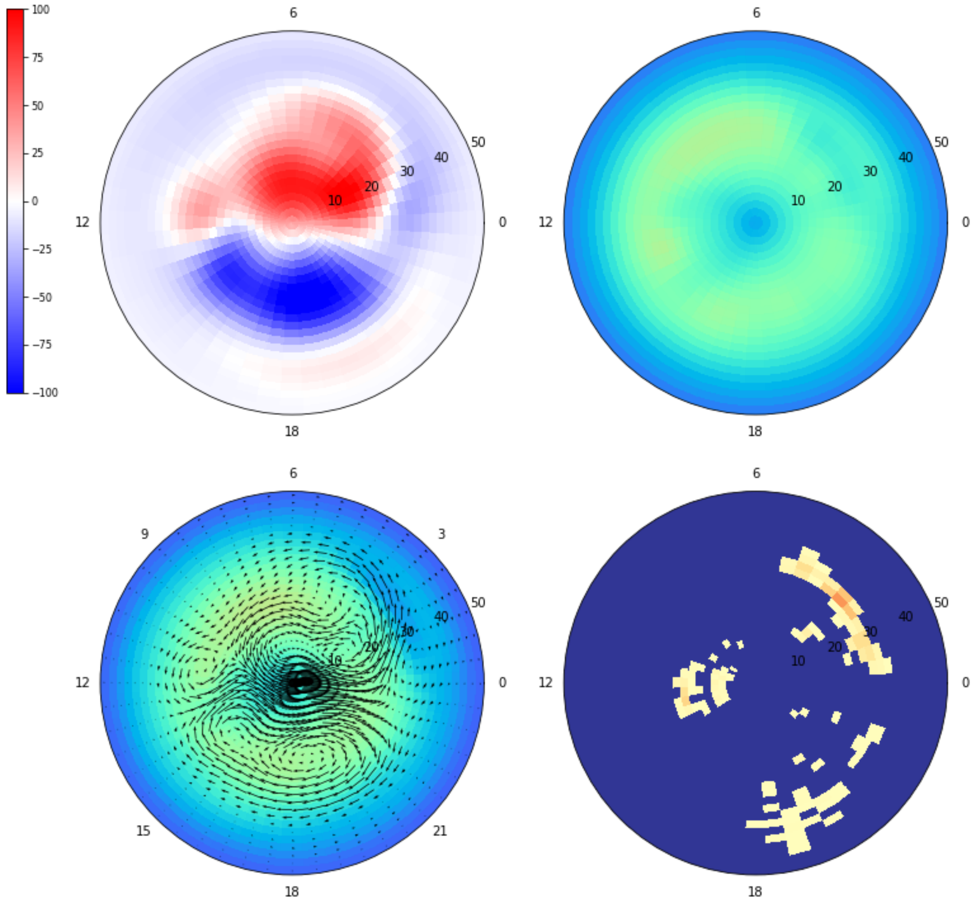
Result



The estimate with actual data

1000 UT, Mar. 27, 2017

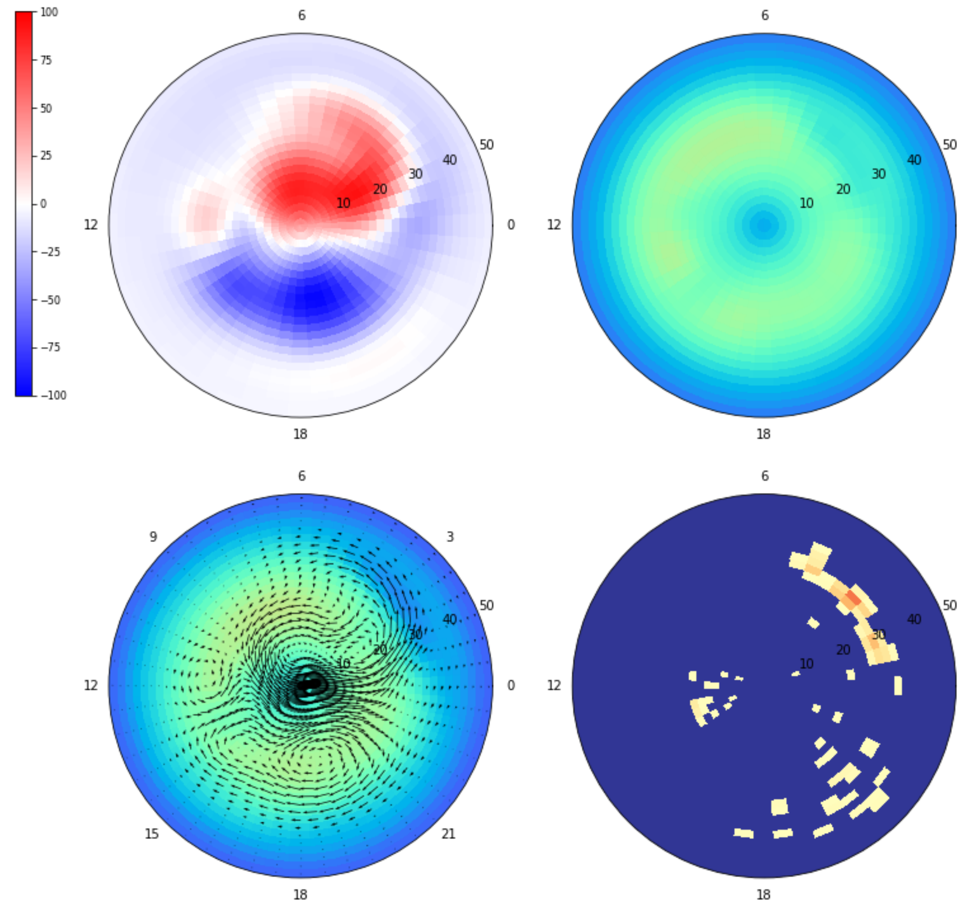
Result



The estimate with actual data

1010 UT, Mar. 27, 2017

Result

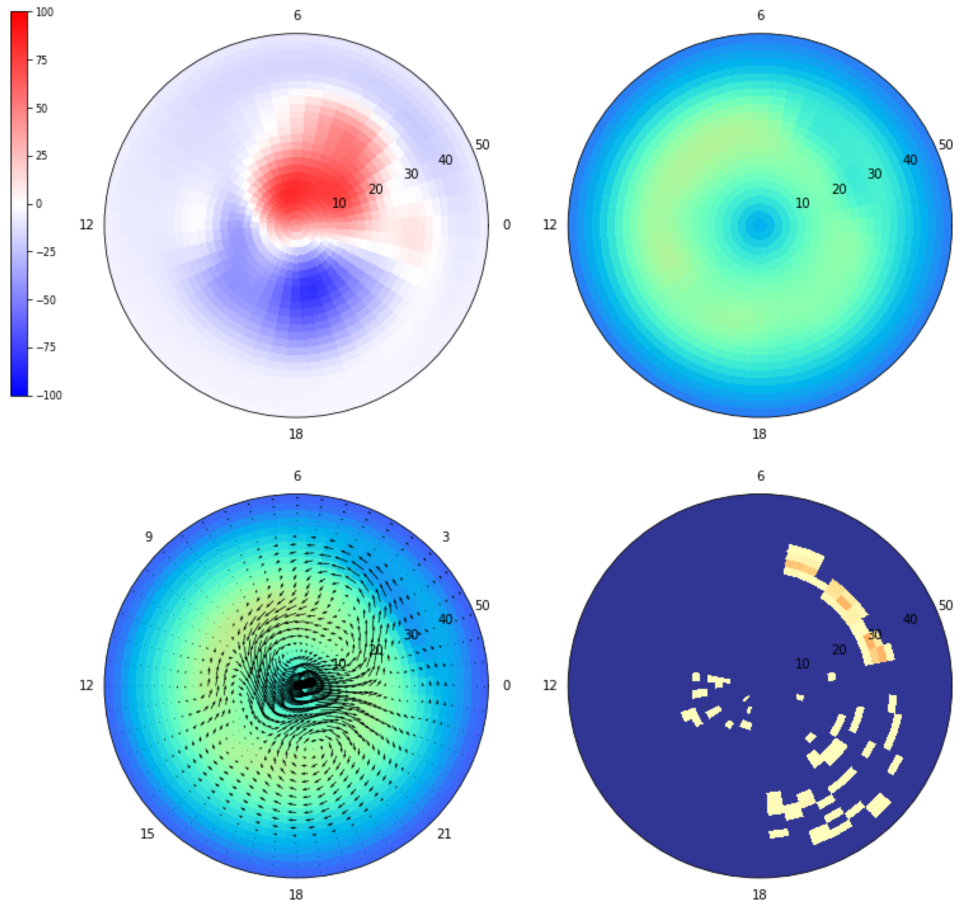


The estimate with actual data

1020 UT, Mar. 27, 2017

Result

The estimate with actual data



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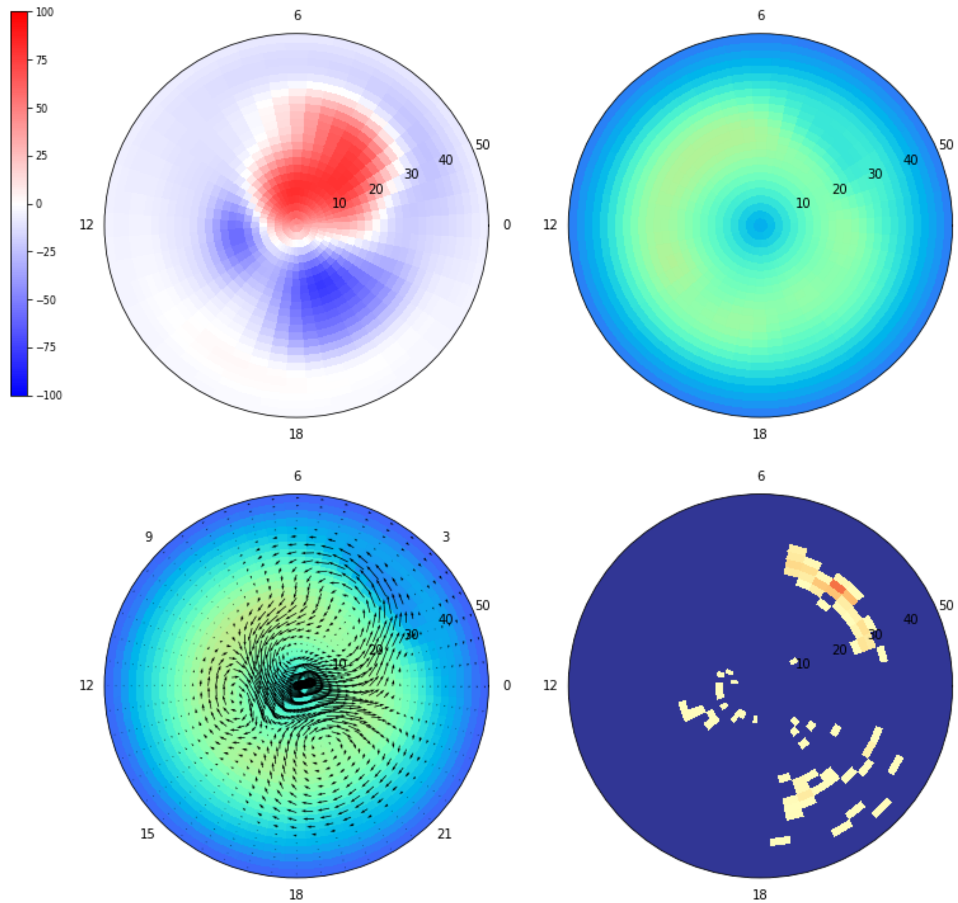
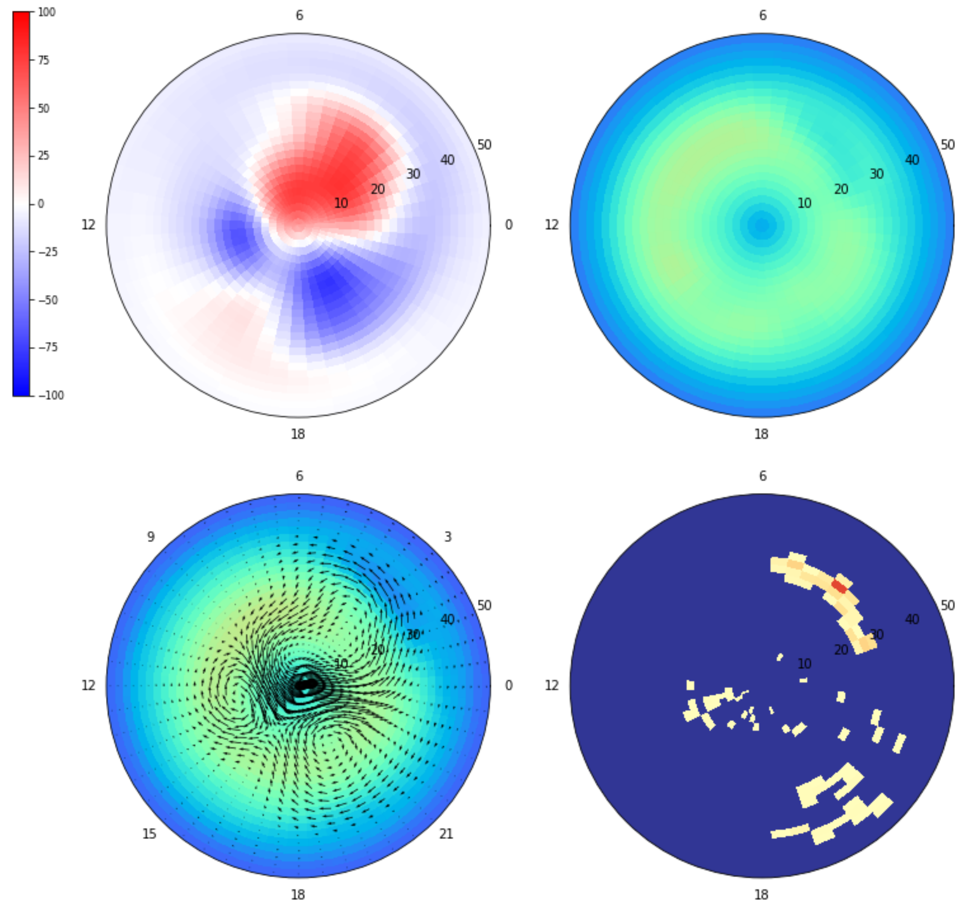


The estimate with actual data

1040 UT, Mar. 27, 2017

Result



The estimate with actual data

1050 UT, Mar. 27, 2017

Summary

- We have proposed a framework for obtaining global flow vector distribution in the ionosphere.
- In our framework, the vector field is represented by weighted sum of localized basis functions derived from a spherical Gaussian function. The basis functions consist of two types of functions: curl-free functions and divergence-free functions.
- Our framework also allows us to combine the SuperDARN data with existing statistical pattern of the velocity distribution.